Calculus Assignment 2

Ans 1.

$$\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw$$

Let $e^{2/w} = t$

$\frac{2}{w} \cdot e^{2/w} = \frac{dt}{dw}$ $\quad \frac{1}{w} dw = \frac{dt}{2e^{2/w}}$

$\Rightarrow \int \frac{1}{w} \times \frac{1}{2e^{2/w}} \cancel{dt}\ dw$ $\quad e^{2/w} = t$

$2/w = \log t$

$w = \frac{2}{\log t}$

$= \int \frac{\log t}{2} \times \frac{1}{2t}\, dt$

$= \frac{1}{4} \int \frac{\log t}{t}$

Let $\log t = v$

$\frac{1}{t} dt = dv$

$= \frac{1}{4} \int v\, dv$

$= \frac{1}{4} \times \frac{v^2}{v} + C$

$= \frac{1}{4} \frac{\log t}{2} + C$

$= \frac{1}{8} \times \frac{4}{w^2} + C$

telco Dt.: Pg.:

$$= \left[\frac{1}{2w^2}\right]_{1/30}^{2}$$

$$= \frac{1}{2}\left[\frac{1}{4} - \frac{1}{(1/30)^2}\right]$$

$$= -\frac{3594}{8}$$

Ans 2. $\displaystyle\int \frac{2t^3+1}{(t^4+2t)^3}\,dt$

Let $t^4+2t^3 = x$

$4t^3+2 = \frac{dx}{dt}$

$$\Rightarrow \frac{1}{2}\int \frac{1}{x^3}\,dx$$

$$= \frac{1}{2}\int x^{-3}\,dx$$

$$= \frac{1}{2}\left(\frac{x^{-2}}{-2}\right) + C$$

$$= -\frac{1}{4x^2} + C$$

$$= -\frac{1}{4(t^4+2t)^2} + C$$

Ans 3.
$$\int f(x)\,dx = \int (x^4 + 3x - 9)\,dx$$

$$f(x) = \int x^4\,dx + 3\int x\,dx - 9\int dx$$

$$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$$

$$f(x) = \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$$

Ans.

Ans 4.

$$f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \le 1 \end{cases}$$

$$\int_{-2}^{3} f(x)\,dx$$

$$\Rightarrow \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$$

$$= \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$$

$$= 3\left[\frac{x^3}{3}\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$$

$$= 21$$

Teleo Dt.: Pg.:

Ans 5. $\int 3(8y-1)e^{4y^2-y}\,dy$

Let $4y^2-y=t$

$(8y-1)\,dy = dt$

$3\int e^t\,dt = 3e^t + C$

Ans $\Rightarrow 3e^{4y^2-y}+C$

Ans 6. $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$f'(1) = -5/4 \quad f(4) = 404$

$\int f''(x) = \int 15\sqrt{x} + 5x^3 + 6\,dx$

$= \frac{15x^{3/2}}{3} + \frac{5x^4}{4} + 6x$

$f'(x) = 10x^{3/2} + \frac{5}{4}x^4 + 6x + C$

$f(1) = 10(1)^{3/2} + \frac{5}{4}(1)^4 + 6(1) \cancel{+C} + C = -\frac{5}{4}$

$C = -\frac{37}{2}$

telco Dt.: Pg.:

$$f'(x) = 10x^{3/2} + \frac{5}{4}x^4 + 6x - \frac{37}{2}$$

$$\int f'(x)\,dx = 10\int x^{3/2}\,dx + \frac{5}{4}\int x^4\,dx + 6\int x - \frac{37}{2}\int dx$$

$$f(x) = 21\,x^{3/2} + \frac{1}{4}x^5 + 3x^2 - \frac{37x}{2} + C$$

~~Ans~~ 7. let $z = f(x+ay) + \phi\,(x-ay)$

differ. w.r.t. x & y

$$\frac{dz}{dx} = f'(x+ay) + \phi'(x-ay) \quad - (1)$$

$$\cancel{dy}\ \frac{dz}{dy} = f'(x+ay)\,a + \phi(x-ay)\cdot a \quad - (2)$$

differ – (1)&(2) w.r.t. x, y respectively

$$\frac{d^2z}{dx^2} = f''(x+ay) + \phi''(x-ay)$$

$$\frac{d^2z}{dy^2} = f''(x+ay)a^2 + \phi''(x-ay)a^2$$

$$= a^2\left[f''(x+ay) + \phi''(x-ay)\right]$$

$$\frac{d^2\cancel{z}}{dy^2} = a^2\,\frac{d^2\cancel{z}}{dx^2}$$

$$\left(\frac{dy}{dx}\right)^2 = 1/a^2 \quad \Rightarrow \quad \frac{dy}{dx} = \frac{1}{a}$$

Ans 9. $\frac{dv}{dt} = 9.8 - 0.196v$

$P = 0.196 \qquad Q = 9.6$

$IF = e^{\int P\,dt}$

$= e^{\int 0.196\,dt}$

$= e^{0.196t}$

$Ve^{0.196t} = 9.8 \int e^{0.196t}\,dt$

$Ve^{0.196t} = 9.8 \left(\frac{e^{0.196t}}{0.196} \right) + C$

$V = 50$

Ans 10. $ty' + 2y = t^2 - t + 1$

$t\left(\frac{dy}{dt}\right) + 2y = t^2 - t + 1$

$\frac{dy}{dt} + \left(\frac{2}{t}\right)y = t - 1 + \frac{1}{t}$

$P = \frac{2}{t}$ & $Q = t - 1 + 1/t$

$If = e^{\int P\,dt}$

$= e^{\int 2/t\,dt}$

$= e^{2\log t}$

$y\,e^{2\log t} = \int \left(t - 1 + \frac{1}{t}\right) \cdot e^{2\log t}\,dt$

Ans 12. $f(x) = x^3 - 10x^2 + 6$

for $x = 3$

Taylor series for $x = a + h$

$$f(a+h) = f(a) + h f'(a) + \frac{h^2 f''(a)}{2!} \dots$$

$f(a) = a^3 - 10a^2 + 6$

$f(3) = -57$

$f'(a) = 3a^2 - 20a$

$f'(3) = -33$

$f''(a) = 6a - 20$

$f''(3) = -2$

$f'''(a) = 6$

$f'''(3) = 6$

$$f(3+h) = f(3) + (x-3)f'(3) + \frac{(x-3)^2 f''(3)}{2!} + \frac{(x-3)^3 f'''(3)}{3!}$$

$$= -57 + (x-3)(-33) + \frac{(x-3)^2(-2)}{2!} + \frac{(x-3)^3(6)}{3!}$$

$$= -57 - 33(x-3) - (x-3)^2 + (x-3)^3$$

— Ans.

Ans 13 $f(a+h) = (1+x)^4$

Using Maclaurien's Series Expansion

$f(a) = (1+a)^4$ $\quad f(0) = 1^4 = 1$

$f'(a) = 4(1+a)^{4-1}$ $\quad f'(0) = 4$

$f''(a) = 4(4-1)(1+a)^{4-2}$ $\quad f''(0) = \cancel{4(4-1)}\ 4(4-1)$

$$f(1+x)^4 = 1 + \left[(1+x)^4\right] + \frac{x\left[(1+x)^4\right]^2 4}{2} + \frac{\left(-(1+u)^{u}\right)^3 4}{3!}$$

Ans

telco Dt.: Pg.:

Ans 15. $f(x) = e^{-6x}$ & $x = -4$

$f(a) = e^{-6a}$ $f(-4) = e^{24}$

$f'(a) = -6e^{-6a}$ $f'(-4) = -6e^{24}$

$f''(a) = 36e^{-6a}$ $f''(-4) = 36e^{24}$

$f'''(a) = -216e^{-6a}$ $f'''(-4) = -216e^{24}$

Taylor Series Expansion

$\Rightarrow e^{24}\left[\frac{1}{6}(x+4) + 3(x+4)^2 + 6(x+4)^3\right]$

Ans 16. $f(x) = \ln(3+4x)$ about $x=0$

$f(a) = \cancel{4/(3+4x)}\ \ln(3+4x)$ $f(0) = \ln 3$

$f'(a) = 4/(3+4x)$ $f'(0) = 4/3$

$f''(a) = -16/(3+4x)^2$ $f''(0) = -16/9$

Taylor Series Expansion

$= \ln 3 + \frac{4}{3}x\left(1 - \frac{2x}{3}\right)$